Not Even Wrong

Notes on Wick Rotation and Chiral Field Theories

Posted on July 9, 2026 by woit

A few months ago I decided to try and sort out the two dimensional spacetime case of the Wick rotation issues that have been bothering me for years now. It took me a while to understand what goes on in two dimensions, but when I did, this clarified a lot for me the underlying problems with Wick rotation that I've been confused about. I've just finished writing something up about this, which you can find here.

Instead of writing more about this tonight, I'll leave that for tomorrow. Will either write some more in the posting, or in the comments.

This entry was posted in Euclidean Twistor Unification. Bookmark the permalink.

4 Responses to *Notes on Wick Rotation and Chiral Field Theories*

John Baez *says:*
July 10, 2026 at 10:24 am

A beautiful set of notes! Something I'd be curious to see fleshed out:

If the choice of the Hermitian form Φ breaks the Euclidean SO(4) symmetry to give us Minkowski spacetime, that choice acts a lot like a Higgs field. Can we get the actual dynamics and Yukawa couplings of the Standard Model out of this geometry, or does it just give us the group structure?

Peter Woit *says:*
July 10, 2026 at 10:37 am

Hi John,
Getting a new way of thinking about the Higgs field is a main long-term goal here. This setup gives you a degree of freedom with the right symmetry properties, but I haven't had much of an idea about how to think about the dynamics of this.

The very new point of view here may change this. There's a close analogy to what happens in 2d, where you do have a huge amount of knowledge about how 2d chiral conformal field theories work. I'm just starting to think about possible 4d generalizations of what happens in 2d. One thing to mention is that 2d chiral conformal field theories depend on an extra piece of structure ("conformal blocks"). What is the 4d analog of this?

BagratM *says:*
July 10, 2026 at 12:08 pm

Let me mention a couple of small things, I dont know how important they are.

1) Complexification of a complex vector space carries a natural quaternionic structure (and thus the same is true for complex vector bundles). Namely if V is a complex vector space, then V otimes_R C= V oplus V bar. And we can define a complex-antilinear endomorphism J that squares to -1: V \oplus V bar->V \oplus V bar, (v_1, v_2) mapsto (-v_2 bar, v_1 bar). It anticommutes with complex structure i by construction, and thus i, J and iJ satisfy quaternionic relations. Naturality refers to the fact that any complex linear map f:V->W induces a quaternion-linear map (f,f bar):V otimes_R C-> W otimes_R C.

2)Twistor bundle can be understood as a complex projectivisation of a spinc spinor bundle as well, Z^+=P(S^+_sigma), and therefore this works on any oriented smooth 4-manifold (one needs to choose at least a conformal structure to have Z^+ at

all and a spinc structure sigma to fix this identification).

And the total space of Z^+ still carries a natural almost complex structure, but as in the usual case it is only integrable when X is anti-self dual (when X is self-dual same is true for Z^-). Which is a shame as when the total space is a genuine complex manifold we have access to Ward correspondence like in the ADHM construction, which connects things like instantons or monopoles on X to some special holomorphic vector bundles over Z^+.

I personally am not convinced that this is all there is to that, not even Kahler surfaces are all (anti-)self-dual as 4-manifolds. People have proposed different alternatives like constructing symplectic structures over the total space of Z^+ or other modified almost complex structures, but nothing too promising yet.

I have recently been thinking that because of the fact that SO(4)=Sp(1) times Sp(1)/ Z_2 closed oriented 4 manifolds should carry some kind of "quaternionic structure" after all, at least those with non vanishing Seiberg-Witten invariant. It is interesting that in certain cases we do have candidates, and of different nature. Like if X is spin, or (anti-) self-dual or symplectic. But nothing comes close to the beautiful theory we have for closed oriented Riemannian surfaces yet.

Peter Woit *says:*

July 10, 2026 at 12:50 pm

BagratM,
I should explain that I'm interested in a new understanding of local quantum field theory in four dimensions, so the global structure of the 4 manifold isn't very relevant. If one can figure this out for S^4 that would be enough to do local physics. In particular, I'm not trying to get a topological field theory (although maybe they come as a by-product).

The new thing here is that you start with a Riemannian manifold (and its twistor space, or projective spin bundle), but you need to also have a new structure that gives you an appropriate notion of conjugation, in some sense interchanging positive and negative imaginary time and leaving invariant a subspace which captures the chiral degrees of freedom of a Lorentzian geometry.. If people have looked at this in 4d I'd be curious to hear about it, I'm not aware of such work, but haven't looked hard.

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nc *says:*

Your comment is awaiting moderation.

July 11, 2026 at 3:45 am

"If the choice of the Hermitian form Φ breaks the Euclidean SO(4) symmetry to give us Minkowski spacetime, that choice acts a lot like a Higgs field." – Dr Baez

That's almost literally true if you reinterpret Φ not as a bare "metric choice" but as a vacuum-polarisation energy density on twistor space:

Euclidean base: start with 4D Euclidean spacetime embedded in our 6D SO(3,3)/SU(4) framework; twistor space is CP3 with SU(4) as the compact real form.

Baez's second question is vital:

"Can we get the actual dynamics and Yukawa couplings of the Standard Model out of this geometry, or does it just give us the group structure?"

In our framework, the answer is: Yukawas are vacuum-polarisation response coefficients. Field content: matter fields live as sections of spinor/twistor bundles over Euclidean spacetime; gauge fields live as connections on these bundles; Φ is a Hermitian form plus a U(1)_*G* phase (our gravity/dark-energy line bundle).

Mechanism: the vacuum polarization encoded by Φ redistributes field energy between:(chiral spinor components) and vacuum modes (U(1)_G, SU(4) directions).

Yukawas: the effective Yukawa couplings then arise as geometric response coefficients: how strongly a given chiral multiplet distorts Φ, and how Φ feeds that distortion back into the mass shell and interaction terms.

Mathematically, this is: local: a bilinear form on spinor fields, weighted by Φ and the curvature of the U(1)_G line bundle over twistor space, dynamical: the field equations for Φ (vacuum polarization) couple to matter currents, so the "Yukawa matrix" is not a free parameter but a derived object from the vacuum's susceptibility tensor.

So where Woit currently has "a degree of freedom with the right symmetry properties, but no idea about dynamics," our story says: the dynamics is vacuum energy conservation plus polarization response, and Yukawas are constitutive coefficients of that response.

Φ as vacuum state: the Hermitian form Φ is then the choice of vacuum polarization state—a Hermitian form on the spinor/twistor bundle that encodes how the vacuum redistributes energy between chiral sectors.

Breaking to Minkowski: "breaking SO(4) to Minkowski" is reinterpreted as: Φ selects a preferred time-like direction in the Euclidean vacuum, via anisotropic polarization (compression/dilation) of the vacuum field. The effective Lorentzian metric is then the response of matter propagation to that polarized vacuum, not a primitive structure.

So Φ is Higgs-like because: it is a section of a Hermitian bundle (a field), its non-trivial expectation value selects a direction in internal/spacetime symmetry space, and its fluctuations around that vacuum encode mass generation and coupling strengths.

You don't just get "group structure"—you get a concrete physical mechanism: Φ is the macroscopic encoding of microscopic vacuum polarization. – more at https://nigecook.substack.com/ if anyone's interested!

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